Logical Arrangement of Topics in the High School Geometry Curriculum: An International Comparison

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Abstract

This paper compares the sequence of topics in geometry courses in the high school curriculum in Texas and Russia. Four textbooks used in Texas and Russia were selected for this comparison.

The order of topics of a course could be arranged in different ways depending on students' learning objectives, preference of the instructor and other factors. The objective of this investigation is to compare the sequence of the topics in the course, determine how the sequences of topics correspond to the purpose of the Geometry course and its expectations, and understand why the topics of the course were arranged in such a way.

Some attention is devoted to the comparison of different definitions of the term "logical" and the usage of this term in this investigation. It turns out this term has a variety of meanings. Almost every author understands the term "logical" in different ways. So, the problem of any educational course is to define a set of certain logical rules which correspond to the purpose of the course.

The author recognizes two approaches in constructing a geometry course:

1) Topic approach – when sections of the book are being arranged in accordance with the complexity of objects/terms under consideration. Such an approach is most suitable for basic education with moderate complexity and expectations.

2) Proof approach – when sections of the book are arranged following proofs of theorems or properties of figures. These types of courses are better suited for more rigorous programs which require a higher-level of complexity and expectations. In Texas, they can be found in Pre-AP, AP and IB curriculum.

Keywords: Geometry, logic, sequence of topics, high school curriculum, objectives of the course, theorems, proofs of theorems.

Logical Arrangement of Topics in the High School Geometry

Curriculum: An International Comparison

Logic is a term used often in our contemporary life. It is used in everyday life and also in the professional and business field, and especially in education. Analysis shows that this term has many definitions, treatments, and way of its usage.

The Merriam Webster Dictionary (1999) gives for term *logic* two basic definitions:

a) A science that deals with the rules and tests of sound thinking and proof by reasoning, and b) Connection in a way that seems reasonable.

In this paper, logic is going to be considered in the "narrow" sense as follows: a set of rules or reasons to select and arrange topics of the geometry course to achieve certain aims and purposes. The first possible rule related to such an approach is that the development of the geometry course should progress from simple objects to more complicated ones. Literal application of such rule directs, for example, the sequence of topics in the following way: A point, a line, an angle, properties of pairs of lines (parallel and perpendicular lines), properties of pairs of angle (angles made by a transversal), triangles, quadrilateral, circles, polygons, and then finally, 3-dimensional figures. The second possible rule for the arrangement of the sequence of topics is the study of properties of objects in the way that every subsequent property follows from the previous ones. As geometric properties of bodies are the result of certain statements or theorems, and such an approach means that every new statement (theorem) would follow from theorems proved before.

The first approach implies that properties of these objects could be learned and taught separately and independently. For example, properties of parallel lines could be taught independently from properties of triangles and properties of triangles could be taught independently from properties of quadrilaterals. It turns out from the practice of geometry that such a point of view is not correct. For example, some properties of parallel lines are based on properties of triangles; in turn, some properties of triangles use properties of parallel lines and quadrilaterals. These issues will be described in detail later in the paper.

Objectives of the Geometry Course and Sequence of Topics

At a contemporary Texas high school in the Houston Independent School District (HISD), the geometry course pursues several purposes. The most important are the following:

- Geometry is aimed to develop common thinking skills and makes students enable apply such skills in the everyday life,
- Students learn basic properties of figures on the plane and in 3D space, and
- Prepare students to the subsequent math courses such as Algebra II, Pre-Calculus, or Trigonometry.

In school districts in Texas, there is a certain list of such purposes or objectives to learn during the school year in each subject. These objectives are described in the Texas

Essential Knowledge and Skills (TEKS); standards accepted for Texas education (TEKS, 2014). The TEKS provide a brief outline of standards and a recommended teaching order for a particular class/grade level. It contains three crucial criteria:

- Student standards that make up one course instruction in the grading cycle,
- Suggested order and objectives for teaching, and
- Recommended number of lessons and amount of time for instructions

So, the teacher's task is to use all these issues in his/her course and create the course which corresponds the needs of students and the educational standards.

In Russia, objectives of the Geometry course are listed in Standards of Education (Russian Portal on General Education. Standards, 2015). This document describes topics and properties of geometric object to learn during every year of education.

Each year the Texas school districts suggest the usage of a certain text book(s). One main (logic) rule used is that the new topic has to follow from the previous ones. In other words, properties of figures have to be derived from previous properties. For this reason sequence of topics (objectives) to be taught in the TEKS has to correlate with the sequence of topics in the text book.

Comparison of Geometry Courses in Texas and Russia

Geometry is the mandatory math course in the math curriculum of both countries. Nevertheless this course is taught in different ways (see Table 1). In Texas, the HISD students, for example, study Geometry only one year after Algebra I course. Usually, they are sophomores (10th grade) or freshmen (9th grade).

Table 1

Differences in the Geometry Courses in U.S.A. (HISD) and Moscow (Russia)

Houston ISD	Moscow School District
1. Geometry course lasts 1 year after Algebra I	1. Geometry course lasts 5 years along with Algebra
2. Students are in 9^{th} or 10^{th} grades	2. Students are in 7^{th} through 11^{th} grades
3. For the most part, text-books have no proofs for statements, theorems, and formulas	3. All statements, theorems, and formulas are proved in the text-book
4. Topics are arranged by geometric objects and properties	4. Topics are arranged by proved properties

In Moscow high schools in Russia, the Geometry course lasts 5 years; it begins at 7th grade and ends at 11th grade. Students study geometry along with algebra. The next difference is that the sequence of topics in textbooks is based on the two different approaches described above. In the textbooks accepted in HISD, topics are arranged by properties of figures regardless the proofs of these properties. In the Moscow school district, the priority is the sequence of proofs and then the sequence of geometric objects.

The sequence of topics in two geometry textbooks used in Houston ISD (McDougal and Little, publisher, 2007) and in the Moscow School District (Prosveshenie, publisher, 2015) are compared in Table 2.

Table 2

Sequence of Topics in Geometry Text Books by McDougal Littlle and Prosveshenie, Publishers

McDougal Littell		Prosveshenie	
Topic	Section	Topic	Section
Main terms, postulates	1.1	Main Terms, Postulates, Segments, Angles	1, 2-16
Segments	1.2	Congruent Triangles;SAS Theorem	15
Angles	1.4	Isosceles Triangles	18
Perimeter and Area	1.7	ASA Theorem	19
Parallel Lines	3.3	SSS Theorem	20
Interior Exterior Angles in Triangles	4.1	Parallel Lines	24
		Relationships Between Sides and Angles In Triangles	33, 34
SSS Postulate	4.3	Right Congruent Triangles (HL)	36
SAS and HL Theorems	4.4	Quadrilaterals and Parallelograms	43, 44
ASA and AAS Theorems	4.5	Area of Rectangles	50, 51
Properties of Isosceles Triangles	4.7	Areas of Other Figures	52, 53, 54
Relationships Within Triangles	5.4	Pythagorean Theorem	55
Pythagorean Theorem	7.1		
Quadrilaterals	8.2		
Areas of Triangles and Parallelograms	11.1		

Main differences in the content in the Table 2 are the following. In the McDougall book, Section 1.7 (area of figures) is after the sections about main terms and segments. Formulas for the area of rectangles and triangles are discussed before these figures are taught in sections 4.1 or 8.2. In the Prosveshenie book, the section about areas stands only in chapters 52, 53, 54 before sections on triangles (Chapter 15), quadrilaterals and parallelograms (Chapters 43 and 44). The reason is that the concept of the area is based on square units (squares) and what's why it is given after properties of quadrilaterals. Formulas for the area of a rectangle are derived from the definition of the area; formulas for parallelograms are based on properties of congruent triangles; and formulas for the triangle is based on the properties of parallelograms. Subsequently, the sections about triangles, rectangles, and parallelograms are to be taught before the concept of the area.

In the McDougal book, statements about congruent triangles are taught as SSS postulate, SAS and HL theorems, and statements ASA/AAS are considered as theorems. These statements are described in Sections 4.3, 4.4, and 4.5. But there are no proofs in the book. Students can learn only to memorize these statements and to apply such statements to solve problems. In the Prosveshenie book all statements about congruent triangles are considered as theorems and all these theorems are proved. The first in this list is the SAS theorem. Following is the section about isosceles triangles and then the SSS theorem, because its proof requires the properties of isosceles triangles. In the McDougal book, Isosceles triangles are learned in the Section 4.7 after congruent triangles.

Traditionally, the Pythagorean Theorem is based on areas of squares erected on sides of the right triangle. So, it looks reasonable to learn this theorem after sections describing properties of quadrilaterals and area of figures. The Pythagorean Theorem is taught in the Prosveshenie text book in Section 55 after Sections 43 about rectangles and Sections 52, 53 on areas of figures. In the McDougal Littell text book, the Pythagorean Theorem is described in section 7.1 before Sections 8.2 and 8.4 about quadrilaterals and Section 11.1, areas of figures. The sequence of topics about Pythagorean Theorem is quite opposite in these two text books.

Now we can compare another pair of geometry text books. It is an American book published by Holt (2007) and another Russian book published by Prosveshenie (referred to as (Prosveshenie II, 2010). The comparison is found in Table 3.

Table 3

Sequence of Topics of Geometry Text Books by Holt and Prosveshnie (II), Publishers

Holt		Prosveshenie II	
Topic	Section	Topic	Section
Points, Lines and Planes	1-1	Points, Segments, Lines	1, 2, 3
Measuring and Segments	1-2	Angles	7
Distance Formula in Plane	1-6	Congruent Triangles: SAS	20
Parallel Lines	3-3	Congruent Triangles: ASA	22
Congruent Triangles: SSS, SAS, ASA		Isosceles Triangles	23
AAS, and HL	4-3	Congruent Triangles: SSS	27
Isosceles and Equilateral Triangles	4-8	Parallel Lines	29
Pythagorean Theorem	5-7	Quadrilaterals and Parallelograms	50, 51
Parallelograms	6-2	Similar Segments	57
Ratios and Similar Polygons	7-2	Pythagorean Theorem	63
Area Formulas for Triangles and		Distance Formula	73
Quadrilaterals	9-1	Area of Figures	120
		Area of Rectangles	122

The main differences in the sequence of topics in these books are the following: In the Holt book, the distance formula in the coordinate plane is in section 1-6. As it is known, this formula is based on the Pythagorean Theorem. The distance formula is nothing as the Pythagorean Theorem expressed in coordinates. But the Pythagorean Theorem stands in

Section 5-7. Moreover, the Pythagorean Theorem itself can be derived either on the basis of the concept of area (as in Section 9) or on the basis of properties of similar right triangles (which are discussed in Section 7-2). In the Prosveshenie II book, the Pythagorean Theorem is derived on the basis of properties of Similar Segments. For this reason Similar Segments are in the Section 60, and the Pythagorean Theorem is discussed in Section 63. In the Prosveshenie II text book, the Distance Formula is in section 73 also after Pythagorean Theorem.

Statements about congruent triangles in the Holt text book are placed in the section 4-3 in the sequence SSS, SAS, ASA, HL. But as already mentioned, proofs of SSS and HL theorems are based on the SAS theorem use properties of Isosceles Triangle which are placed in

Section 4-8, after the section 4-3. In the Prosveshenie II book, SAS theorem goes first (Section 20), before the ASA theorem (Section 22) and before section 23, isosceles triangles. SSS is then discussed in Section 27.

The list of such comparison could be extended with other geometric objects and their properties. The logical rules of the arrangements in above mentioned text books will be considered.

Discussion and Conclusions

It is clear that the text books have different approaches (i.e.,. logical rules) for the arrangement of topics. In books used by HISD topics are arranged by objects, regardless of proofs. In books used by Moscow school districts, the accent is made on sequence of proofs. It is worth mentioning that these books are used in geometry courses of different duration. In HISD, students study the subject for only one year. For this reason there is not much opportunity to derive properties of geometric objects successively. The main focus is made on the *application* of formulas and properties for calculation of different measures (i.e., lengths, areas, or volumes). The majority of problems in the McDougall Littell and Holt books relate to calculation aspects. Once proofs are not the center of the course, then it is more affordable for students to learn topics arranged in accordance with geometric objects (from simple to more complicated) and learn geometric properties separately. Such an approach is quite reasonable and demonstrates its validity. On the other hand, the books published by Prosveshenie are used over a five year course. Such duration gives more opportunities to make the accent of proofs of properties and illustrates how these properties follow from to another. The calculation aspects such as lengths, areas, and volumes are addressed later.

In conclusion, geometry text-books such as those published by McDougal Little and Holt are convenient to use in short courses. They do not require deep analysis of geometric properties and can be easily memorized by students. An abundance of calculation problems help to develop practical skills for application of learned properties in the life. Geometry text-books such as those published by Prosveshenie publisher, are more suitable for extended courses. They pay more attention to foundations (not basics) of the subject and its logical interrelations. They are appropriate for higher level courses such as Pre-AP and AP level.

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