

Pattern Blocks, Area, and Content Acquisition

James R. Valles, Jr.
Prairie View A&M University
Mathematics
jrvalles@pvamu.edu

Rebecca Ortiz
Texas Tech University
Advertising

This study examined the results of a pre-test and a post-test, as well as a post-test survey, that covered problems focusing on non-unit areas represented by pattern blocks. The solutions presented are examined for dependence by the pre-service teacher participants to rely on algebraic means to solve the pictorial problems as well as looking into their experience working with manipulatives. Their results, looked at in the context of manipulative use in the United States and internationally, is discussed.

INTRODUCTION

Representations can be difficult for pre-service math teachers to grasp and for new teachers to use in their classroom. Even and Tirosh (1995) state that “many new teachers do not have a solid understanding of the subject matter they teach” (pg. 6). Part of the rationale for integrating representations into the teaching of math is to provide a bridge for students to connect a mathematical concept to their personal lives (i.e. to help students own the knowledge). As Hill and Ball (2009) point out, the desire to help students connect math to their lives is more complicated mathematically than it may initially appear. And while representations can be used in a class, Karp (2010) says that teachers have trouble using representations meaningfully in their teaching.

Despite the difficulties inherent in the use of representations, their use is encouraged, and they are vital to students’ success in learning mathematics. The National Council of Teachers of Mathematics (2000) emphasizes conceptual understanding, strategic competence, and adaptive reasoning. As Griffin, Jitendra, and League (2009) state, the traditional aims of math learning are important, but now “students must develop well-connected conceptual knowledge as a foundation underlying mathematical procedures” (pg. 319). This conceptual knowledge is in part what the use of representations is meant to illuminate to students.

Stylianou (2010) said that representations are central to providing understanding of a mathematical concept and to a person’s problem-solving ability. Thus representations should not be taught and learned as if they were the ends as opposed to a means, since this has been hypothesized as a contributing factor to students’ difficulties in math (NCTM, 2000). Representations should instead be an essential tool

in mathematical teaching. Ball, Sleep, Boerst, and Bass (2009) state: “Skilled mathematics teaching requires more than simply learning how to enact particular pedagogical tasks. It also requires knowing and using mathematics in ways that are distinct from simply doing math oneself” (pg. 461).

DEFINING MANIPULATIVES

No clear definition of a manipulative exists in the literature. Goldin (2002) states that “a representation is a configuration that can represent something else in some manner” (pg. 208). Manipulatives have also been defined as physical objects used as teaching tools for the purpose of learning math in a hands-on manner (Smith, 2009). Hynes (1986) defined manipulatives as “concrete models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around by students” (p. 11).

This definition of manipulatives needs revision, however, when dealing with virtual manipulatives. Moyer, Bolyard, and Spikell (2002) define a virtual manipulative as “an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (pg. 373). Even agreement on a representation is open to interpretation. The NCTM (2000) states “the term representation refers both to process and product—to the act of capturing a mathematical concept or relationship in some form and to the form itself” (p. 67).

What comes out of this collection of terminology is the consensus that mathematical activity is dependent upon various forms of representations. Students need to be exposed to numerous methods of visualizing mathematical ideas so that they develop a library of mental images which leads to a deeper abstract understanding

of a concept (Moyer, 2001). Based off of the NCTM definition of a representation, both the images developed from work with manipulatives as well as the process of working with the manipulative are vital to that student understanding.

Among the literature available discussing work with manipulatives is the handling of pre-service teachers as well as in-service teachers working with and understanding how to use manipulatives. Elementary students who have teachers who integrate manipulatives into their classroom can positively benefit from such inclusion (Boggan, Harper, & Whitmire, 2010). But training future teachers to be comfortable with manipulatives can be stress-inducing in itself; for some, the task of learning to use manipulatives also involves relearning mathematics as well (Vinson, 2001).

USING MANIPULATIVES

The use of manipulatives in a mathematics lesson presents a challenge to teachers. Students who use manipulatives during instruction typically demonstrate a deeper understanding of the concept modeled (Boggan, Harper, & Whitmire, 2010; Goldin, 2002; Caglayan, 2013). However, this does not imply that manipulatives are a magic key to unlocking a student's understanding of a mathematical concept. Moyer (2001) discusses how teachers may tend to use manipulatives during what was characterized as "fun math". Uttal, Scudder, and DeLoache (1997) state that while manipulatives can be effective in teaching math, the manipulative in itself does not possess the key to knowledge, as this depends on how the children interpret the manipulative and relate it back to the concept presented. Thus there should be a two-prong focus on introducing and using manipulatives with pre-service teachers: helping

them relearn mathematics with the benefit of manipulatives and helping them learn how to deploy manipulatives in their classroom lessons.

What this paper will examine is the initial knowledge and acquisition of area as an abstract concept through the use of pattern blocks and the self-efficacy of a group of pre-service teachers related to this exercise. Furthermore, we will discuss how this relates to findings on a global perspective and whether what was attempted in this experiment can shed any insight on what has been previously discussed in the literature both from an American perspective as well as an international perspective.

METHODS AND PROCEDURES

Twenty-one pre-service teachers enrolled in a mathematics content course designed for pre-service teachers were participants in this study. This content course was the second of a two-course sequence at a Texas university. The data from the twenty-one students are those who participated in both a pre-test and post-test administered. Between the pre-test and the post-test, the students were given an assignment where they were to use virtual pattern blocks to determine area values based on defined areas having specified values. For example, one such question asks

$$\text{If } \color{blue}{\text{trapezoid}} + \color{red}{\text{trapezoid}} = 1, \text{ then } \color{green}{\text{triangle}} = \underline{\hspace{1cm}}?$$

One question (#2) on the assessment asks

$$\text{If } \color{blue}{\text{trapezoid}} + \color{red}{\text{trapezoid}} = 3, \text{ what is } 1\frac{1}{5}?$$

The data collected is from the pre-test and the post-test as well as a post-test questionnaire. The answers provided to the pattern block problems were examined for the correctness of the answers provided as well as for the method in which the solutions were determined. The questionnaire results were also examined with regard to previous

experience using pattern blocks, how the pattern blocks were used (if at all), and whether or not the pre-service teachers believed they could be used to help teach fractions.

FINDINGS

What we found among the posed problems was a lack of experience working with pattern blocks that was among the biggest obstacles faced by the participants. Among those who indicated they had used manipulatives of any kind prior to this study, many of the responses indicated that manipulatives had been used to provide a visual for rather concrete examples (e.g. using circles to learn about fractions, using base-10 blocks, using counters to learn about adding and subtracting numbers). There was little indication that problems of the type posed in this study (i.e. representations of non-unit areas) had ever been explored with any type of manipulative. Most exposure to manipulatives as a learning tool was limited to either elementary school or (less frequently) to other college pre-service teacher preparation courses.

Regarding pattern blocks, three students specifically discussed how they remembered playing with pattern blocks but not with respect to any particular mathematical concept. Using the pattern blocks to represent areas that were non-unit values presented difficulties to the students. Of the problems posed, those that had a higher frequency of correct responses were those where the triangle represented either one-half or one-fifth. Some of the students attempted to find the solution by subdividing the shapes given into smaller parts. As one student commented on Question 2, "I can't think of anything smaller than a triangle in order to answer this question, which is extremely frustrating." She made a similar comment on another question, stating "I want

to subdivide like I do with music, but with shapes, but I can't think of a smaller shape like I know smaller notes. I feel like I am almost there, but not quite getting it."

This search for smaller subdivisions created, within the responses, a subset of responses of students who attempted to find the solutions through algebraic means.

The intent of the problems posed was not to lead the students to consider the possible algebraic solutions. However, as was stated in the post-test surveys as well as on the pre-test and post-test, there were instances where algebraic means were utilized in an attempt to find a solution. One student solved the problems algebraically, for example using ▲ in place of a variable such as x . This was reflected in one student's response to how he would teach Question 2 in the study:

Divide into triangles. Each triangle (▲) is equal to x , an unknown value. Find x , then how many of x go into the final value. Alternatively, you may set up a

proportion $\frac{5}{3} = \frac{x}{\frac{6}{5}}$ where x is the number of triangles and the denominator is the

corresponding value.

What the post-test showed, when looking at what concerns the students had and how they envisioned teaching a fractions lesson, what that many students felt they needed remediation themselves on some level. Nineteen of the students expressed different levels of lack of understanding or confidence in working with fractions, ranging from needing some extra assistance of their own to wanting to see more lessons involving manipulatives to feeling completely lost. Five of these students, however, were able to answer two of the three fractions correctly on the post-test, thus indicating they

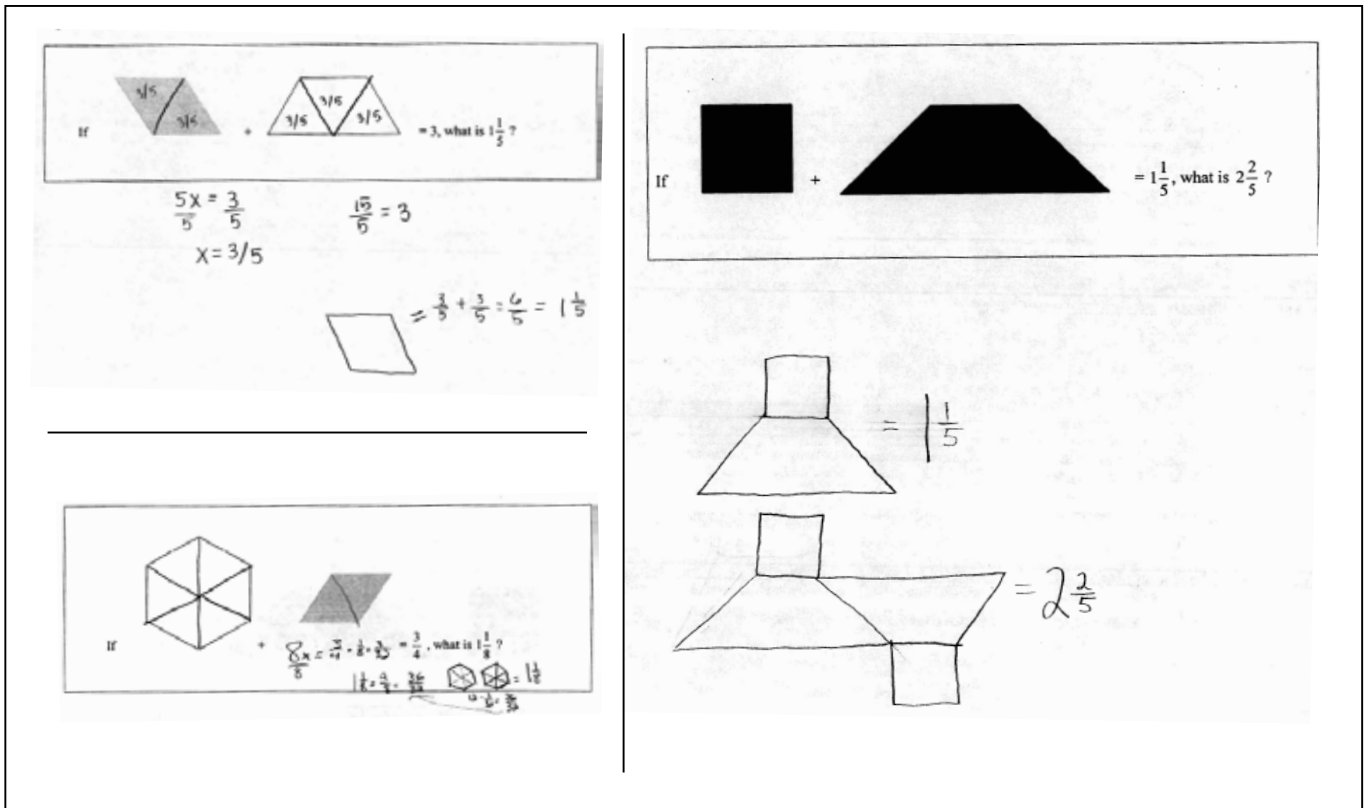


Figure 1: Examples of responses submitted on the post-test.

were comfortable with the questions given but perhaps not with extending the fractional area concept to manipulatives.

On the survey, ten students indicated that they remembered using manipulatives in elementary school. Of these, two stated that they remember using manipulatives as play objects; one of the students said “I remember using things like pattern blocks in first grade. Sadly, I only really remember playing with them during spelling tests because I got a 100 on the pre-test.”

DISCUSSION AND CONCLUSIONS

The use of representations by teachers is one aspect of teaching which is difficult to impart onto new teachers specifically because the key concepts that representations are meant to illuminate are not typically encompassed by the knowledge of math most educated people acquire. Teaching math requires more of the teacher than simply knowing the mathematical material, but rather the teacher must possess the knowledge

of how to teach the concept as well. Representations are one method that, when used effectively, can help the teacher bridge math concepts to the students.

As educators of teachers, we need to be aware of the limitations pre-service teachers have in both their mathematical knowledge and the teaching knowledge of mathematics. Using manipulatives for the sake of having them used should not be the goal of a lesson. Instead, there need to be clearly-defined goals of both the students and the teacher in charge of the lesson.

Beswick, Callingham, and Watson (2011) state that the belief of mathematics as computation may persist in teachers despite the development of knowledge to teach math. The use of manipulatives can certainly help bridge the gap between computation and the abstract nature of concepts, but it is reasonable to ask how this should be accomplished. One study indicates that manipulatives, when used in math instruction, tend to produce small- to medium-sized effects on student learning when compared to instructions that utilized only abstract symbols (Carbonneau, Marley, & Selig, 2013).

In considering the global implications of the use of manipulatives in mathematics teaching, a number of differing views have to be accounted for. Stevenson and Stigler (1992) point out that Japanese teachers would use the same types of manipulatives throughout elementary school, as opposed to using a variety of materials as American teachers do, so as to prevent confusion among the students as to what the manipulatives are to represent. Naidoo (2011) discusses how manipulatives that proved effective in helping students conceptualize topics through the use of concrete manipulatives. Lai and Murray (2012), in examining deep understanding of math among Chinese students as compared to American students, state that multiple applications of

a method, thus allowing for the introduction of visual representations of problems that are interrelated, can provide students a scaffold for seemingly unrelated applications of concepts. One study involving Lebanese fifth-grade students concludes with a recommendation that multiple modes of representations are best for stimulating student learning as well as for teachers having a tool to expose where students are having issues with mathematical concepts (Chahine, 2013). Within the past thirty years, there has been a noticeable change in the increased use of manipulatives in Finnish mathematics classes, but this is only one part of the Finnish curriculum system (Pehkonen, 2009).

This collection of results points to some possibilities of how to proceed with using manipulatives. One conclusion is realizing that utilizing manipulatives in mathematics lessons will almost certainly involve more work on the part of the teacher, in the forms of developing understanding of them and in working with them. Proper deployment of said manipulatives is thus crucial, but not necessarily as only a reward for good classroom behavior – they should be part of the rigorous study. Based on the results of this study in working with pre-service teachers, they should be encouraged to not rely on or fall back on former algebraic knowledge to work with manipulatives, lest the concept of what the manipulatives provide be lost. In this vein, though, the use of manipulatives in lessons needs to be developed accordingly and not off of previously-acquired knowledge. It is in this line of thought to pose as a future research question whether the use of strictly one, or at least a minimum number, manipulatives would provide more beneficial results than the use of an assortment of different types of manipulatives. While it can be seen that manipulatives in and of themselves are not the only solution to

delivering quality mathematical content and teaching, the best practices and implementations associated with hands-on concrete representations seem to mirror the same issues associated with manipulatives.

References

- Ball, D. L., Sleep, L., Boerst, T. A., & Bass, H. (2009). Combining the development of practice and the practice of development in teacher education. *The Elementary School Journal*, 109(5), 458 – 474.
- Beswick, K., Callingham, R., & Watson, J. (2012). The nature and development of middle school mathematics teachers' knowledge. *Journal of Mathematics Teacher Education*, 15(2), 131-157. doi: 10.1007/s10857-011-9177-9
- Boggan, M., Harper, S., & Whitmire, A. (2010). Using manipulatives to teach elementary mathematics. *Journal of Instructional Pedagogies*, 3, 1-6.
- Caglayan, G. (2013). Prospective mathematics teachers' sense making of polynomial multiplication and factorization modeled with algebra tiles. *Journal of Mathematics Teacher Education*, 16(5), 349-378. doi: 10.1007/s10857-013-9237-4
- Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal Of Educational Psychology*, 105(2), 380-400. doi:10.1037/a0031084
- Chahine, I.C. (2013). The impact of using multiple modalities on students' acquisition of fractional knowledge: An international study in embodied mathematics across semiotic cultures, *The Journal of Mathematical Behavior*, 32(3): 434-449.
<http://dx.doi.org/10.1016/j.jmathb.2013.04.004>.
- Even, R. & Tirosh, D. (1995). Subject-matter knowledge and knowledge about students as sources of teacher presentations of the subject-matter. *Educational Studies in Mathematics*, 29(1), 1–20.

- Goldin, G. A. (2002). Representation in mathematical learning and problem solving. In L. English (Ed.), *Handbook of international research in mathematics education* (pp. 197–218). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.
- Griffin, C. C., Jitendra, A. K., and League, M. B. (2009). Novice special educators' instructional practices, communication patterns, and content knowledge for teaching mathematics. *Teacher Education & Special Education, 32*(4), 319 – 336.
- Hill, H., & Ball, D. L. (2009). The curious – and crucial – case of mathematical knowledge for teaching. *Phi Delta Kappan, 91*(2), 68 – 71.
- Hynes, M., (1986). Selection criteria. *Arithmetic Teacher, 33*(6), 11–13.
- Karp, A. (2010). Analyzing and attempting to overcome prospective teachers' difficulties during problem-solving instruction. *Journal of Mathematics Teacher Education, 13*(2), 121 – 139.
- Lai, M. Y., & Murray, S. (2012). Teaching with procedural variation: A Chinese way of promoting deep understanding of mathematics. *International Journal for Mathematics Teaching & Learning*. Retrieved from <http://www.cimt.plymouth.ac.uk.ezproxy.pvamu.edu/journal/lai.pdf>
- Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies In Mathematics, 47*(2), 175-197.
- Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What are virtual manipulatives? *Teaching Children Mathematics, 8*(6), 372.
- Naidoo, J. (2011). Scaffolding the teaching and learning of mathematics. In Venkat, H. and Essien, A. (Eds.), *Proceedings of the Seventeenth National Congress of the*

- Association for Mathematics Education of South Africa* (pp. 124-134). Retrieved from <http://www.amesa.org.za/AMESA2011/Volume1.pdf>
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Pehkonen, E. (2009). How Finns learn mathematics: What is the influence of 25 years of research in mathematics education? In Lepik, M. (Eds.), *Proceedings of the 10th International Conference* (pp. 71-101). Retrieved from http://www.tlu.ee/bcmath2009/Tallinn%202009_proceedings.pdf
- Smith, S.S. (2009). *Early childhood mathematics* (4th ed.). Boston: Pearson Education.
- Stevenson, H.W. & Stigler, J.W. (1992). *The learning gap: Why our schools are failing and what we can learn from Japanese and Chinese education*. New York: Summit Books.
- Stylianou, D. (2010). Teachers' conceptions of representation in middle school mathematics. *Journal of Mathematics Teacher Education*, 13(4), 325 – 343.
- Uttal, D.H., Scudder, K.V., & DeLoache, J.S. (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of Applied Developmental Psychology*, 18, 37-54.
- Vinson, B. (2001). A comparison of preservice teachers' mathematics anxiety before and after a methods class emphasizing manipulatives. *Early Childhood Education Journal*, 29(2), 89-94.